Trigonometrikus és hiperbolikus azonosságok

Trigonometrikus azonosságok

$$\cos^2 x + \sin^2 x = 1; \quad \sin(k\pi) = 0; \quad \cos(k\pi) = (-1)^k; \quad \sin(k\pi + \frac{\pi}{2}) = (-1)^k; \quad \cos(k\pi + \frac{\pi}{2}) = 0;$$

$$\cos(x+\pi) = -\cos x; \quad \sin(x+\pi) = -\sin x; \quad \cos(x+2\pi) = \cos x; \quad \sin(x+2\pi) = \sin x;$$

$$\cos(\pi - x) = -\cos x; \quad \sin(\pi - x) = \sin x; \quad \cos(-x) = \cos x; \quad \sin(-x) = -\sin x;$$

$$\sin(\frac{\pi}{2} - x) = \cos x; \quad \sin(\frac{\pi}{2} + x) = \cos x; \quad \cos(\frac{\pi}{2} - x) = \sin x; \quad \cos(\frac{\pi}{2} + x) = -\sin x;$$

$$\sin(x+y) = \sin x \cos y + \cos x \sin y; \quad \sin(x-y) = \sin x \cos y - \cos x \sin y;$$

$$\cos(x+y) = \cos x \cos y - \sin x \sin y; \quad \cos(x-y) = \cos x \cos y + \sin x \sin y;$$

$$\sin 2x = 2 \cos x \sin x; \quad \cos 2x = \cos^2 x - \sin^2 x = 2\cos^2 x - 1 = 1 - 2\sin^2 x;$$

$$\operatorname{tg}(x+y) = \frac{\operatorname{tg} x + \operatorname{tg} y}{1 - \operatorname{tg} x \operatorname{tg} y}; \quad \operatorname{tg}(x-y) = \frac{\operatorname{tg} x - \operatorname{tg} y}{1 + \operatorname{tg} x \operatorname{tg} y}; \quad \operatorname{tg} 2x = \frac{2 \operatorname{tg} x}{1 - \operatorname{tg}^2 x}$$

$$\operatorname{ctg}(x+y) = \frac{\operatorname{ctg} x \operatorname{ctg} y - 1}{\operatorname{ctg} x + \operatorname{ctg} y}; \quad \operatorname{ctg}(x-y) = \frac{\operatorname{ctg} x \operatorname{ctg} y + 1}{\operatorname{ctg} y - \operatorname{ctg} x}; \quad \operatorname{ctg} 2x = \frac{\operatorname{ctg}^2 x - 1}{2 \operatorname{ctg} x}$$

$$\cos^2 \frac{x}{2} = \frac{1 + \cos x}{2}; \quad \sin^2 \frac{x}{2} = \frac{1 - \cos x}{2}; \quad \operatorname{tg} \frac{x}{2} = \frac{\sin x}{1 + \cos x}$$

$$\cos x \cos y = \frac{\cos(x+y) + \cos(x-y)}{2}$$

$$\sin x \sin y = \frac{\cos(x-y) - \cos(x+y)}{2}$$

$$\sin x \cos y = \frac{\sin(x+y) + \sin(x-y)}{2}$$

$$\sin x + \sin y = 2 \sin \frac{x+y}{2} \cos \frac{x-y}{2}; \quad \sin x - \sin y = 2 \cos \frac{x+y}{2} \sin \frac{x-y}{2}$$

$$\cos x + \cos y = 2 \cos \frac{x+y}{2} \cos \frac{x-y}{2}; \quad \cos x - \cos y = -2 \sin \frac{x+y}{2} \sin \frac{x-y}{2}$$

Hiperbolikus trigonometrikus azonosságok

$$\operatorname{ch} x = \frac{e^x + e^{-x}}{2}; \quad \operatorname{sh} x = \frac{e^x - e^{-x}}{2}; \quad \operatorname{ch}^2 x - \operatorname{sh}^2 x = 1$$

$$\operatorname{th} x = \frac{\operatorname{sh} x}{\operatorname{ch} x} = \frac{e^{2x} - 1}{e^{2x} + 1} = \frac{1 - e^{-2x}}{1 + e^{-2x}}; \quad \operatorname{cth} x = \frac{\operatorname{ch} x}{\operatorname{sh} x} = \frac{e^{2x} + 1}{e^{2x} - 1} = \frac{1 + e^{-2x}}{1 - e^{-2x}}$$

$$\operatorname{sh}(x+y) = \operatorname{sh} x \operatorname{ch} y + \operatorname{ch} x \operatorname{sh} y; \quad \operatorname{sh}(x-y) = \operatorname{sh} x \operatorname{ch} y - \operatorname{ch} x \operatorname{sh} y;$$

$$\operatorname{ch}(x+y) = \operatorname{ch} x \operatorname{ch} y + \operatorname{sh} x \operatorname{sh} y; \quad \operatorname{ch}(x-y) = \operatorname{ch} x \operatorname{ch} y - \operatorname{sh} x \operatorname{sh} y;$$

$$\operatorname{sh} 2x = 2 \operatorname{ch} x \operatorname{sh} x; \quad \operatorname{ch} 2x = \operatorname{ch}^2 x + \operatorname{sh}^2 x = 2 \operatorname{ch}^2 x - 1 = 1 + 2 \operatorname{sh}^2 x;$$

$$\operatorname{th}(x+y) = \frac{\operatorname{th} x + \operatorname{th} y}{1 + \operatorname{th} x \operatorname{th} y}; \quad \operatorname{th}(x-y) = \frac{\operatorname{th} x - \operatorname{th} y}{1 - \operatorname{th} x \operatorname{th} y}; \quad \operatorname{th} 2x = \frac{2 \operatorname{th} x}{1 + \operatorname{th}^2 x}$$

$$\operatorname{cth}(x+y) = \frac{1 + \operatorname{cth} x \operatorname{cth} y}{\operatorname{cth} x + \operatorname{cth} y}; \quad \operatorname{cth}(x-y) = \frac{1 - \operatorname{cth} x \operatorname{cth} y}{\operatorname{cth} x - \operatorname{cth} y}; \quad \operatorname{cth} 2x = \frac{\operatorname{cth}^2 x + 1}{2 \operatorname{cth} x}$$

$$\operatorname{ch}^2 \frac{x}{2} = \frac{\operatorname{ch} x + 1}{2}; \quad \operatorname{sh}^2 \frac{x}{2} = \frac{\operatorname{ch} x - 1}{2}; \quad \operatorname{th} \frac{x}{2} = \frac{\operatorname{sh} x}{\operatorname{ch} x + 1} = \frac{\operatorname{ch} x - 1}{\operatorname{sh} x}; \quad \operatorname{cth} \frac{x}{2} = \frac{\operatorname{ch} x + 1}{\operatorname{sh} x} = \frac{\operatorname{sh} x}{\operatorname{ch} x - 1}$$

$$\operatorname{ar\,sh} x = \log\left(x + \sqrt{x^2 + 1}\right); \quad \operatorname{ar\,ch} x = \log\left(x + \sqrt{x^2 - 1}\right);$$

$$\operatorname{ar\,th} x = \frac{1}{2} \log \frac{1+x}{1-x}; \quad \operatorname{ar\,cth} x = \frac{1}{2} \log \frac{1-x}{1+x}$$

$$\vdots$$